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Research Article

An Agent-Based Model of a Pricing Process with Power Law, Volatility Clustering, and Jumps

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In this paper, we propose a new model of security price dynamics in order to explain the stylized facts of the pricing process such as power law distribution, volatility clustering, jumps, and structural changes. We assume that there are two types of agents in the financial market: speculators and fundamental investors. Speculators use past prices to predict future prices and only buy assets whose prices are expected to rise. Fundamental investors attach a certain value to each asset and buy when the asset is undervalued by the market. When the expectations of agents are exogenously driven, that is, entirely shaped by exogenous news, then they can be modeled as following a random walk. We assume that the information related to the two types of agents in the model will arrive randomly with a certain probability distribution and change the viewpoint of the agents according to a certain percentage. Our simulated results show that this model can simulate well the random walk of asset prices and explain the power-law tail distribution of returns, volatility clustering, jumps, and structural changes of asset prices.

1. Introduction

Many empirical studies point out the fact that there are typically power-law tails, volatility clustering, jumps, and structural changes in time series of financial asset prices. Gabaix et al. [1] summarized many research results and found heavy-tailed long-range distributions with characteristic power-law exponents, the so-called inverse cubic law. They pointed out that this is rather "universal" for financial markets in most countries, with time intervals ranging from one minute to one month, across different sizes of stocks, different time periods, and different stock market indices. Later, they proposed a basic model with power-law distribution in which volatility is caused by the trades of large institutions [2]. The volatility clustering means that the price changes of financial assets are positively correlated. The high volatility or low volatility of the stock market tends to concentrate in a certain period of time, presenting the so-called volatility clustering effect.

In recent years, the jumps in security price have also attracted widespread attention. Jump-diffusion models are considered in Merton [3]. Eraker [4] and others found that, in real financial markets, stock price data cannot be fully explained by Brownian motion. They named the behavior of the asset price change in a short period of time the "price jump behavior". The jumps in price are generally believed to be due to the release of news and liquidity [5, 6]. At the same time, the phenomenon of structural changes based on price jumps is also the focus of research. The structural changes of volatility is ubiquitous in the stock markets of various countries, which makes people pay more attention to the stability of stock market volatility and reasons that lead to volatility changing structure. In general, the appearance of the structural changes of volatility is caused by some big events in the social economy. The violent volatilities of the stock market may be caused by the adjustment of economic policies or the influence of some other major economic events, such as financial innovation, failure of internal control mechanism of enterprises, failure of financial risk relationships, the out of control of the market supervision mechanism, etc. These factors often make stock price ticket deviate from the normal fluctuation, and bring shock to the financial system. Eraker, Johannes, and Polson [7] have definitive evidence that the volatility of returns is affected by structural mutations, and they point out that there are structural changes in both volatility and returns. Broadie, Chernov, and Johannes [8] also reach similar conclusions. It is generally believed that the long-memory property of asset price changes is closely related to structural changes [9, 10].

Based on the typical characteristics of the above asset pricing processes, Inoua [11] proposed a random walk model that reflects the changes in asset prices. However, this model only reflects the power law and volatility clustering of the pricing process. There is no discussion of the jumps and structural changes. Hence, based on the work of Inoua [11], we have improved the model in order to reflect the jumps and structural changes of asset prices.

The remainder of this paper is organized as followings. In Section 2, we describe some typical facts about the process of asset price in China's stock market, such as the power-law characteristics of the distribution of returns, volatility clustering, jumps in prices, and structural changes. In Section 3 we propose an improved price dynamics model based on the work of Inoua [11]. The fourth section contains the simulation analysis. By analyzing a simulation of the pricing process, we determine the typical dynamic characteristics of asset prices. In the last section, we present our conclusions.

2. Empirical Analysis of Asset Price Changes

Below we give an empirical example of typical characteristics of the pricing process in China's stock market. The data we use comes from a total of 58,560 pieces of high-frequency transaction data of Shanghai Stock Exchange (SSE) 50 index stocks (SSE 50 Index is the stock index of Shanghai Stock Exchange, representing the top 50 companies by "float-adjusted" capitalization), from January 3, 2017, to December 29, 2017. Since China's stock market has only 4 hours of trading time per trading day, the overnight effect and alternate-day effect may exist if all 1-minute price series are directly connected. This will affect the results of the jump test on asset prices, so we need to address the overnight effect and alternate-day effect first. The overnight returns after processing are given by

$$r_{Gt} = \left(\frac{r_{gt} - \overline{r_{gt}}}{\sqrt{s}} \right) \sigma + \mu \quad (1)$$

where r_{gt} denotes the original overnight returns, $\overline{r_{gt}}$ denotes the mean of r_{gt}, s denotes the variance of all the samples, and σ and μ are the standard deviation and mean of all nonovernight returns. The treatment of the alternate-day effect is the same as for the overnight effect.

Let P_t be the price of a financial asset at time t, and $r_t \equiv (P_t - P_{t-1})/P_{t-1}$ be the return during this period. According to [1, 2, 12], the returns follow the power law distribution with cubic exponent. That is,

$$P\left(\left|r_t\right| > x\right) \sim Cx^{-\mu} \quad (2)$$

as $x \longrightarrow \infty$, where $\mu \approx 3$ and $C > 0$.

We analyze the one-minute price series as an example. Figure 1 displays the results concerning price series, return series, power-law distribution, and autocorrelation.

To test the volatility clustering effect of returns, we use the Breusch-Godfrey method, which is based on the idea of Lagrange multiplier testing, to test heteroscedasticity. The basic idea of this method is to build the auxiliary function between the residual squared sequence and the explanatory variables and to obtain the regression sum of squares (ESS), in order to judge the significance of heteroscedasticity. For a given level of significance of 0.05, the p-value of the return is 2.2e-16. Thus, we can reject the null hypothesis that the residual variance is constant and infer that the heteroskedasticity does exist. This confirms the volatility clustering property. In short, Figure 1 shows that asset price series follow a random walk, and the tail of the distribution of returns exhibits a power-law distribution with an exponent of 3. The volatility of asset prices shows a significant long-memory property.

We use the LM test proposed by Lee and Mykland [13] to detect jump arrival times on the high-frequency returns. The core concept of this method is to construct a statistic based on local volatility and instantaneous volatility and use it to deduce the distribution function. As long as the significance level is given, by calculating the value of the statistic corresponding to the stock price at a certain moment and comparing it with the threshold of the significance level, it can be judged whether this point is a jump. We process the 1-minute return series at a sampling interval of 5 minutes and apply the LM test. We find that there are 32 jumps between January 3 and December 29, 2017. The specific jumps are shown in Figure 2.

To further illustrate the structural changes of the asset pricing process, we analyze the data of 58,560 1-minute closing prices. We first perform natural logarithm processing on the selected 58,560 pieces of data and obtain a total of 58,559 pieces of data after the first order difference. Then, the CUSUM test is used to detect the structural changes of the 1-minute return series, which is based on nonparametric estimation [14–16]. The results are shown in Figure 3.

As seen from Figure 3, the returns of the SSE 50 index show a significant volatility clustering effect and structural change characteristics (the dashed line in the figure).

On the basis of the analysis above, we can infer that there are typically power-law tail distribution, volatility clustering, jumps, and structural changes in stock index returns. These typical facts of the asset-price processes cannot be explained by traditional financial theory. Some scholars have tried to establish more complex theoretical models to explain these facts, such as the theory of fractal markets proposed by Peters [17], and some scholars have used agent-based modeling to build a new theoretical framework [18]. However, as Inoua [11] pointed out, these facts are widespread in empirical analysis and there must be some basic rules that account for them. This paper improves the random walk model proposed by Inoua [11] to reflect these facts that are typical of the asset pricing process.

3. Price Dynamics Model

As a typical complex dynamic system of multiagent interaction, financial market has attracted extensive attention and made great progress in the past decades. Quantifying

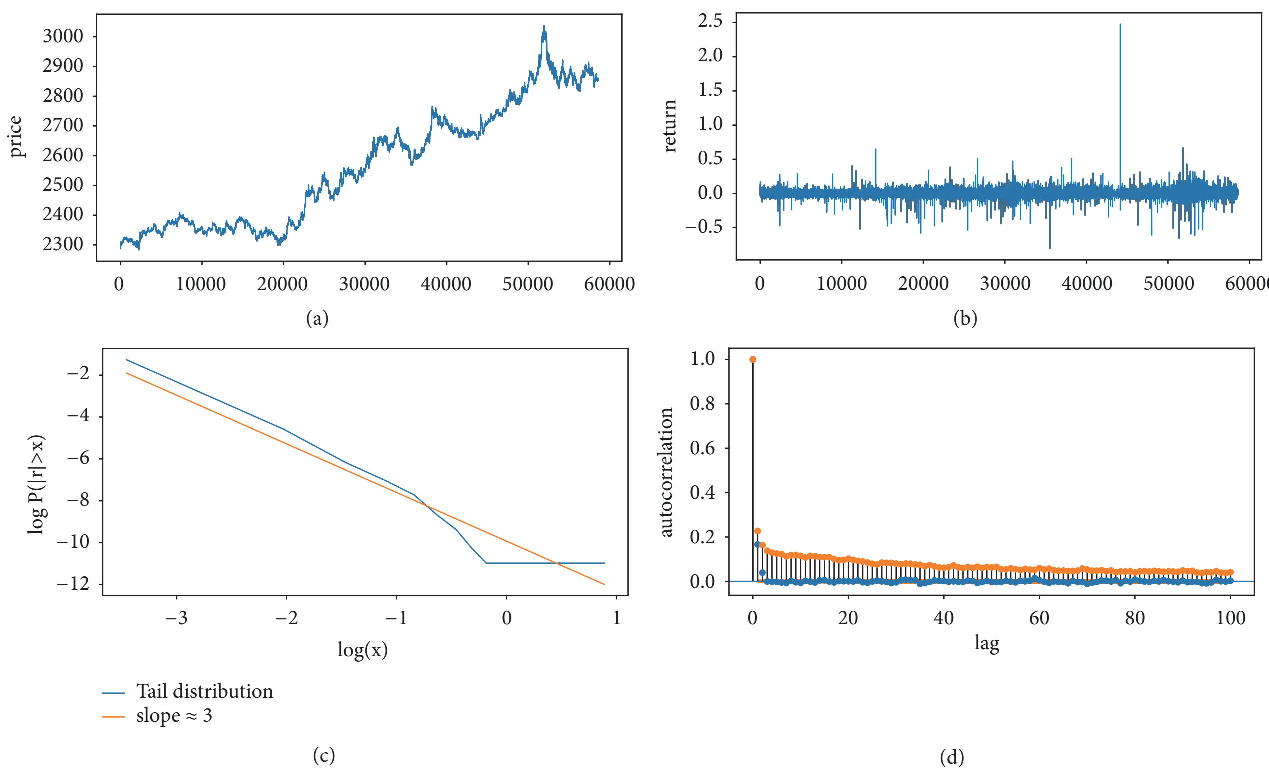

Figure 1: *The empirical analysis of the SSE 50 Index's 1-minute price series.* (a) 1-minute price series; (b) return series that eliminates the overnight effect (in percentage); (c) tail distribution of the absolute return in the log-log scale and a least-square fit for values with a slope of three; (d) autocorrelation function of return and absolute return, where the autocorrelation function of absolute return decays slowly, indicating that the volatility has long-memory characteristics.

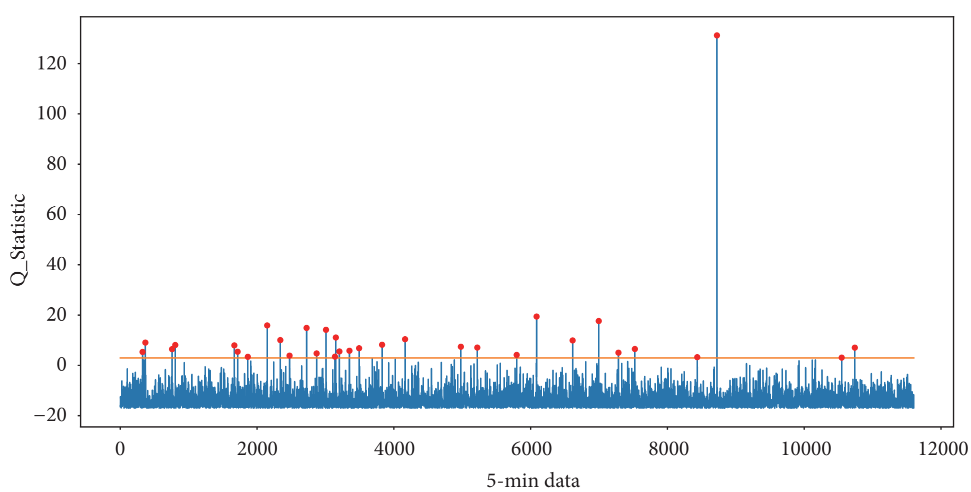

Figure 2: *Jumps of the 5-minute interval under the LM test.* Eliminate the overnight effect of the 2017 SSE 50 index data. Process the data at a 5-minute sampling interval and calculate the value of the LM statistic. Compared with the yellow horizontal reference line for the significance level threshold in the figure, the 32 points marked red are beyond the normal value range.

price dynamics in financial markets will provide a great foundation for deepening our understanding of financial market behaviors. There have been various approaches to study the performance of financial markets. Poggio et al. [19] use agent-based model of financial markets to match those in experimental-market settings with human subjects and simulate complex interactions among artificial intelligence traders with varying degrees of learning capabilities. Biondo [8] introduce a new Self-Organized Criticality (SOC) model for simulating price evolution in an artificial financial market, based on a multilayer network of traders. Lima and Miranda [20] use the Itô's stochastic differential equation for the

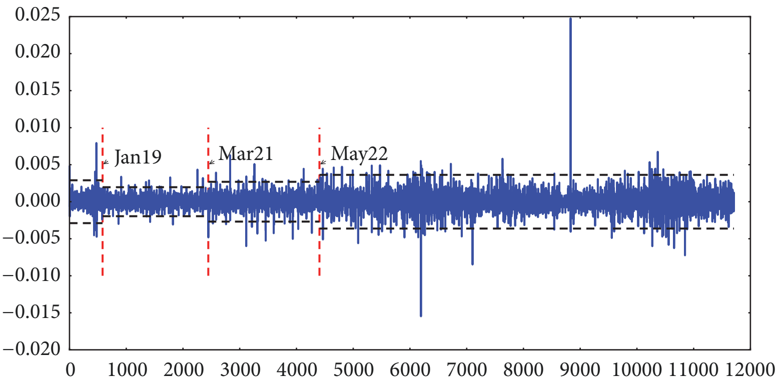

Figure 3: *Changes in the 1-minute returns in the Chinese Stock Market*. The CUSUM test finds 3 structural changes, which can divide the entire return series into 4 phases. The first phase is from January 3 to January 18, the second phase is from January 19 to March 20, the third phase is from March 21 to May 21, and the fourth phase is from May 22 to December 29.

double well with additive white noise as a mathematical model for price dynamics of the financial market.

According to Provenzano [21], most of these agent-based models consist of two typical types of agents. The first type of agent considers that the stock price is determined by the fundamental market value of the asset, while the second type of agent can predict the future price using simple trading rules, extrapolation of trend, and patterns observed in past prices. In the power-law distribution model proposed by Inoua [11], the main players of the market are speculators. They use past prices to predict future prices, form price expectations, and only buy assets whose prices are expected to rise (otherwise, they sell). Since the price expectations of the market are endogenous, the return in the speculative market obeys the autoregressive process of random coefficients. Later, Inoua [22] added fundamental investors to the original model in order to achieve the volatility clustering of the pricing process. Fundamental investors attach a certain value to each asset and choose to buy or sell assets based on their judgment of the value of the assets. The model further assumes that the expectations of the two types of trading agents are affected by exogenous information at the same time; the price is considered to be obeying the random walk process with exogenous impacts, which can imply volatility clustering in a generic way.

Our model is based on the above model. We assume that the market consists of two types of agents: speculators, who buy an asset for a purely speculative reason, that is, when they expect its price to rise, and sell otherwise; and fundamental investors, who attach a certain value to an asset and buy it when they think it is worth more than its current price, and sell otherwise. The expected returns for these two types of agents are affected by the external information (event flow) of the same market, but the two types of agents differ in their response intensity.

On the basis of Inoua [11] model, we assume that the demand of the ith speculator is $x_{it} = \alpha r_{it}^{(1)}$, where $r_{it}^{(1)}$ denotes the returns that the ith speculator expects to gain in period t, and the superscript (1) indicates that the agent is of the first type, namely, a speculator. We assume that the demand of the jth fundamental investor is $x_{jt} = \gamma r_{jt}^{(2)}$, where $r_{jt}^{(2)}$ denotes the returns that the jth speculator expects to gain in period t, and the superscript (2) indicates that the agent is of the second type, namely, a fundamental investor. And $\alpha, \gamma > 0$ are constant conversion factors, indicating that the agent's demand for assets changes linearly with the expected return of the asset at a certain ratio.

The two types of agents have different estimated expected returns. For speculators, the expected return is $r_{it}^{(1)} = (\widetilde{P_{it}} - P_{t-1})/P_{t-1}$, where $\widetilde{P_{it}}$ is the price of the ith speculative estimate in period t. For fundamental investors, the expected return is $r_{jt}^{(2)} = (V_{jt} - P_{t-1})/P_{t-1}$, where V_{jt} is the asset value of the jth fundamental investor's estimates in period t. The total demand can be expressed as

$$x_t = \alpha N_t \overline{r_t^{(1)}} + \gamma M_t \overline{r_t^{(2)}} \tag{3}$$

where N_t and M_t denote the number of speculators and fundamental investors, respectively, who desire to buy or sell an asset during period t. Then, the expected returns for all speculators and fundamental investors are, on average, $\overline{r_t^{(1)}} \equiv N_t^{-1} \sum_i r_{it}^{(1)}$ and $\overline{r_t^{(2)}} \equiv M_t^{-1} \sum_j r_{jt}^{(2)}$, respectively.

We suppose that the return changes linearly with the total demand; this can be written as $r_t = \beta x_t = \alpha\beta N_t \overline{r_t^{(1)}} + \gamma\beta M_t \overline{r_t^{(2)}}, \beta > 0$. To simplify the calculation, let $a_t \equiv \alpha\beta N_t$ and $b_t \equiv \gamma\beta M_t$; then the return is written as

$$r_t = a_t \overline{r_t^{(1)}} + b_t \overline{r_t^{(2)}} \tag{4}$$

Inoua [11] pointed out that when $E(a_t) < 1$ and $E(b_t) < 1$, (4) is the Kesten [23] process, and there is a power-law distribution with an exponent greater than 1. The dynamics of price can then be calculated by using the following equation:

$$P_t = P_{t-1}(r_t + 1) = P_0 \prod_{k=1}^{t} (r_k + 1) \tag{5}$$

Hence, the price will depend on the composition of the two types of agents: those with endogenous expectations and those with exogenous expectations.

Suppose the impact of external information (event flow) on individual decisions is E_t. E_t can be generally expressed as a semimartingale process, such as a typical diffusion-jump process, where the diffusion component represents the comprehensive influence of daily market information, and the jump component can be a typical compound Poisson process indicating that the market is impacted by large external events. In order to simplify the analysis, this paper assumes that E_t is an independent and identically distributed normal process synchronized with the pricing process. As we can see in the simulation section, this simple assumption can still obtain various typical characteristics of the return series. We use the indicator function, $I(|E_t| \geq \theta_1)$ and $I(|E_t| \geq \theta_2)$, to indicate that when the impact of external events reaches θ_1 and θ_2, respectively, it will have a correspondingly large impact on the expected returns of speculators and fundamental investors. If we assume the expectations of the two types of agents follow the AR(3) process, then we have

$$\overline{r_t^{(1)}} = \varphi_1 \overline{r_{t-1}^{(1)}} + \sigma_1 E_t I\left(|E_t| \geq \theta_1\right) \tag{6}$$

$$\overline{r_t^{(2)}} = \varphi_2 \overline{r_{t-1}^{(2)}} + \sigma_2 E_t I\left(|E_t| \geq \theta_2\right) \tag{7}$$

where φ_1 and φ_2 are autoregression coefficients and σ_1 and σ_2 are scale factors, which are used to adjust the magnitude of the external events' impact value and reduce the influence of the extreme values. In particular, when $\varphi_1 = \varphi_2 = 1$, (6) and (7) are random walk processes.

By using the random walk model, that is, letting $\varphi_1 = \varphi_2 = 1$ in (6) and (7), the expression of the return becomes

$$\begin{aligned} r_t = {} & a_t \left[\overline{r_0^{(1)}} + \sum_{k=1}^{t} \sigma_1 E_k I\left(|E_k| \geq \theta_1\right)\right] \\ & + b_t \left[\overline{r_0^{(2)}} + \sum_{k=1}^{t} \sigma_2 E_k I\left(|E_k| \geq \theta_2\right)\right] \end{aligned} \tag{8}$$

where $\overline{r_0^{(1)}} = 0$ and $\overline{r_0^{(2)}} = (\overline{V_0} - P_0)/P_0$ and $\overline{V_0}$ denotes the initial expected value of the asset for the fundamental investor.

If the reaction of the two types of agents to the impact of external events on the market changes over time, that is, if agents' expectations are related to market conditions, such as a bear or bull market, then θ_1 and θ_2 can be regarded as functions of time or market conditions. The return can then be written as

$$\begin{aligned} r_t = {} & a_t \sum_{k=1}^{t} \sigma_1 E_k I\left(|E_k| \geq \theta_1(t)\right) \\ & + b_t \left[\overline{r_0^{(2)}} + \sum_{k=1}^{t} \sigma_2 E_k I\left(|E_k| \geq \theta_2(t)\right)\right] \end{aligned} \tag{9}$$

or

$$\begin{aligned} r_t = {} & a_t \sum_{k=1}^{t} \sigma_1 E_k \begin{cases} I\left(|E_k| \geq {\theta_1}^{(I)}\right) & 1 \leq k < m \\ I\left(|E_k| \geq {\theta_1}^{(II)}\right) & m \leq k \leq t \end{cases} \\ & + b_t \left[\overline{r_0^{(2)}} \right. \\ & \left. + \sum_{k=1}^{t} \sigma_2 E_k \begin{cases} I\left(|E_k| \geq {\theta_2}^{(I)}\right) & 1 \leq k < m \\ I\left(|E_k| \geq {\theta_2}^{(II)}\right) & m \leq k \leq t \end{cases}\right] \end{aligned} \tag{10}$$

where ${\theta_1}^{(I)}$ and ${\theta_2}^{(I)}$ denote the threshold when the market is in state(I) and ${\theta_1}^{(II)}$ and ${\theta_2}^{(II)}$ denote the market's threshold under state(II).

By adjusting the thresholds under different market conditions, we can also determine the structural changes caused by the impact of exogenous information.

4. Simulation Analysis

In this chapter, we will simulate the price dynamics model (10) derived in the previous section. We first set the initial asset price and value as 100 and carry out 10,000 time unit simulation of the price change process according to the established model. We hypothesize that the expected returns of the two types of agents would follow the random walk process in (6) and (7), and the thresholds and other parameters involved in the random walk process are given by (10). Here we assume that E_t is an independent and identically distributed standard normal distribution sequence synchronized with the pricing process, which can be directly generated by random numbers. The result of each step of the random walk will have an impact on the subsequent trend of asset price and value, which will further affect the expectations of both types of agents. Finally, we will get the asset price, value, and return sequences based on the random walk process of two types of agents. The specific parameter settings and simulation results are analyzed as follows.

4.1. Parameter Settings. In the price dynamics model (10), $\sigma 1$ and $\sigma 2$ are the scaling parameters. Due to the fact that influence of external events E_t follows the standard normal distribution, which is different from the expected returns of the two types of investors in the numerical values, we refer to the method in Inoua's paper and adjust the impact values of external events by introducing scale factors. Therefore the expected returns will not have a large change or even a negative value, which is consistent with the actual situation of the market. Referring to the parameter settings of Inoua [11], we let the scale factors $\sigma 1$ and $\sigma 2$ be 0.001 and 0.1. According to the settings in the paper, the probability of the events associated with the arrival in period t of events relevant to the two types of agents are 1 and 0.1, respectively. In the new mechanism we construct, the threshold θ_1 for the occurrence of the event that has an impact on the speculator is 0, indicating the inevitable event. And the range of the

Table 1: Sensitivity analysis of threshold θ_2.

θ_2	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Mean of price	99.93	100.22	100.11	100.09	99.64	99.91	100.17	100.14	99.89	100.13
Std. of price	2.84	2.78	2.68	2.55	2.36	2.27	2.11	1.93	1.84	1.72
Skewness	-0.03	-0.01	-0.01	-0.03	-0.01	-0.01	-0.03	0.02	-0.02	-0.01
Kurtosis	-0.50	-0.55	-0.55	-0.53	-0.53	-0.52	-0.50	-0.50	-0.51	-0.56
Mean of the power exponent	3.17	3.17	3.18	3.17	3.17	3.18	3.16	3.17	3.17	3.18
Std. of the power exponent	0.23	0.23	0.23	0.22	0.22	0.23	0.23	0.23	0.23	0.23

Table 2: Sensitivity analysis of the parameter of exponential distribution a_t.

a_t	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
Mean	100.11	100.02	100.02	99.86	99.64	100.19	100.16	99.81	100.08	99.90
Std.	2.61	2.57	2.48	2.42	2.36	2.34	2.29	2.26	2.25	2.19
Skewness	-0.02	0.01	0.01	-0.01	-0.01	0.01	0.04	-0.04	-0.01	-0.01
Kurtosis	-0.59	-0.56	-0.61	-0.54	-0.53	-0.51	-0.46	-0.39	-0.38	-0.29
Mean of the power exponent	3.11	3.14	3.14	3.16	3.17	3.19	3.20	3.20	3.20	3.22
Std. of the power exponent	0.21	0.23	0.22	0.23	0.22	0.22	0.24	0.24	0.23	0.23

Table 3: Sensitivity analysis of the parameter of exponential distribution b_t.

b_t	0.26	0.27	0.28	0.29	0.30	0.31	0.32	0.33	0.34	0.35
Mean	99.90	99.84	100.10	100.02	99.64	100.07	100.17	100.12	100.08	99.90
Std.	2.29	2.33	2.33	2.40	2.36	2.38	2.41	2.40	2.25	2.19
Skewness	-0.01	-0.01	-0.01	0.01	-0.01	0.01	-0.01	0.01	0.01	-0.01
Kurtosis	-0.50	-0.52	-0.52	-0.53	-0.53	-0.52	-0.56	-0.51	-0.51	-0.29
Mean of the power exponent	3.23	3.19	3.19	3.17	3.17	3.16	3.16	3.15	3.15	3.12
Std. of the power exponent	0.23	0.24	0.23	0.22	0.22	0.22	0.22	0.23	0.23	0.23

threshold θ_2 of the fundamental investor is adjusted to make the corresponding event happen with a small probability of about 3% to 15%, then the sensitivity analysis is performed as shown in Table 1.

By adjusting θ_2, we can see that, as its value increases, the standard deviation of price decreases, having little effect on other statistical indicators. Within this range, we may select a value of 1.5 with a probability of about 7% for the following simulation.

Inoua using exponential distributions for a_t and b_t with respective means 0.1 and 0.3 and verifies that the model holds for different distributions of a_t and b_t and for a broad range of values of the parameters. Due to the improvement of the mechanism, in order to make the simulation results more consistent with the empirical data, we adjust the parameters of exponential distribution a_t and b_t on the basis of Inoua's model and carry out sensitivity analysis. The results are shown in Tables 2 and 3.

By adjusting the value range of a_t, we find that, as the value increases, the standard deviation of price decreases and the power exponent of the power-law distribution increases. And as the parameter of b_t increases, only the power exponent of the power-law distribution decreases, having little effect on other statistical indicators. In order to get closer to the empirical data on the basis of the original model, we refer to the parameter range of the original model and select the parameter values of 0.05 and 0.3 in the following simulation, respectively

As described by Inoua [11], the volatility clustering is generic in this model, and it holds for different distributions of a_t and b_t and for a broad range of values of the parameters (as we have checked). Most parameters play only a quantitative role; therefore, parameters σ_1, σ_2, θ_1, and θ_2 are chosen in this simulation merely to 'calibrate' the model with the real daily data displayed in Figure 1, namely, to have a standard deviation of return around one percent. Next, we simulate on the basis of the above parameter settings and analyze the results of the simulation.

4.2. Power-Law Distribution and Volatility Clustering of Pricing Process. In the parameter settings in the previous section, we use exponential distributions for a_t and b_t with respective means of 0.05 and 0.3. Let the scale factors σ_1 and σ_2 be 0.001 and 0.1, respectively, and the threshold values θ_1 and θ_2 of the external information impact be 0 and 1.5, respectively. Let the initial asset price and value both be 100, and simulate 10,000 time units of the price-changing process according to our model. The results are shown in Figure 4.

To see clearly the changing process of price and value, we selected the first 500 sets of data for the simulation results in Figure 4(a); they indicate that the trend of prices is basically the same as the change of value. Figure 4(b) includes 10,000

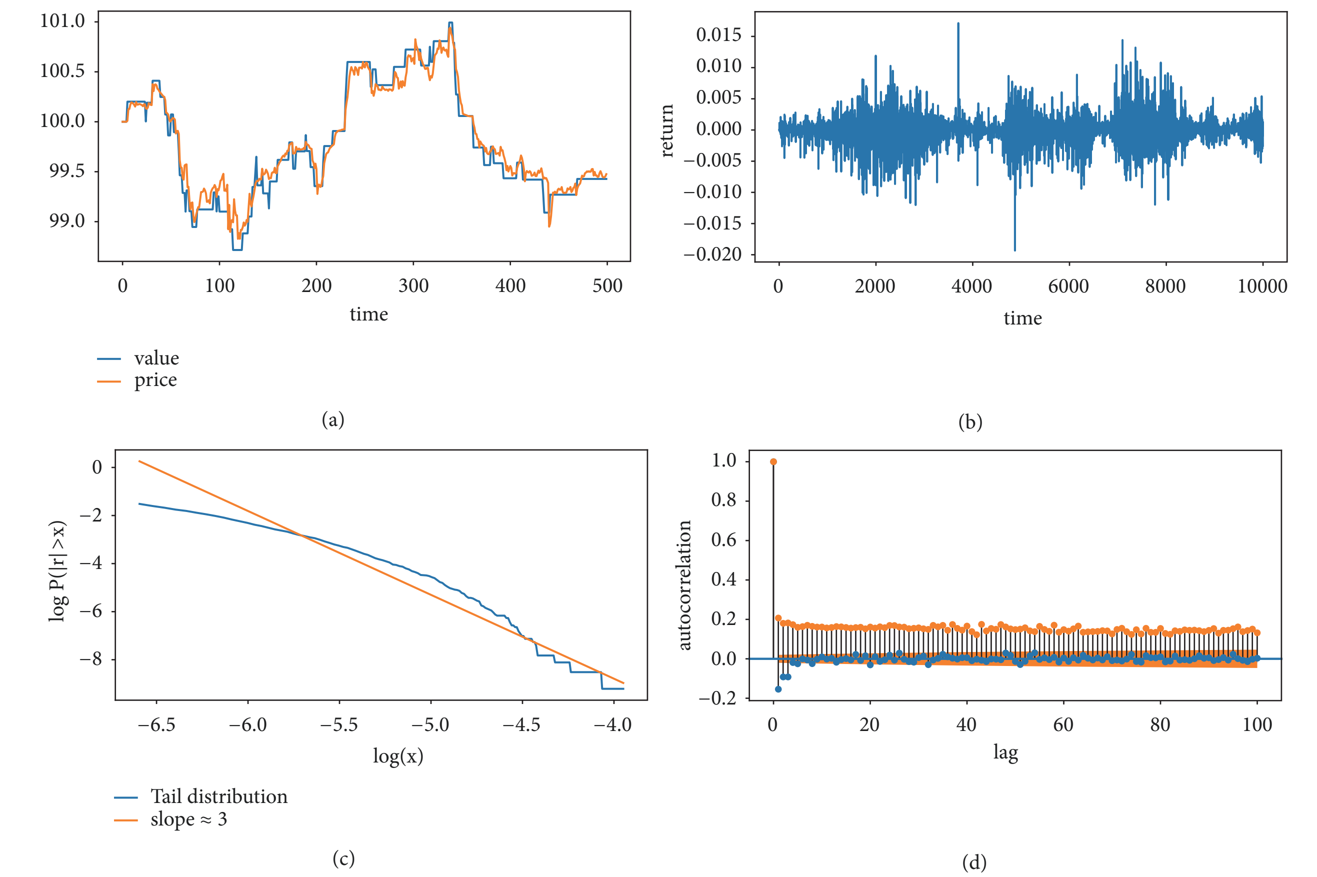

Figure 4: *Simulation results*. (a) Simulated trajectory of asset prices and values (for clarity, only the 500 first sets of data are shown); (b) return series (in percentage); (c) tail distribution of the absolute return in the log-log scale and a least-square fit for values; (d) autocorrelation function of return and absolute return; the autocorrelation function of absolute return decays slowly, indicating that the volatility has long-memory characteristics.

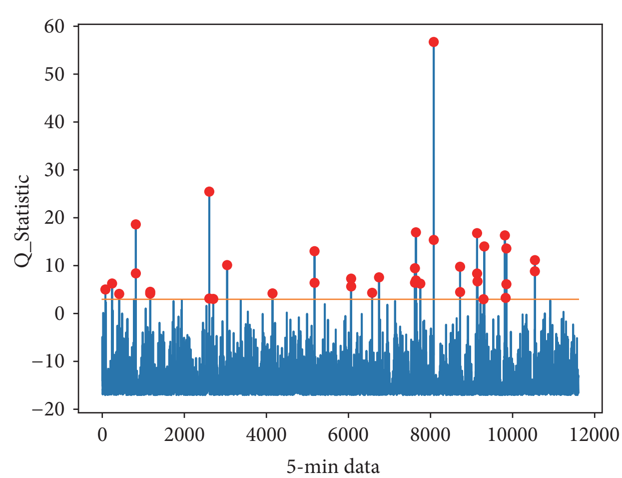

Figure 5: *Jumps of the simulated data of the 5-minute interval under the LM test*. Referring to the length of the empirical data, we simulate the price data of 58560 time units. Process the data at a 5-minute sampling interval and calculate the value of the LM statistic. Compared with the yellow horizontal reference line for the significance level threshold in the figure, the 32 points marked red are beyond the normal value range.

sets of return data, which show that there is a significant volatility clustering effect. The power exponent of the power-law distribution in Figure 4(c) is 3.2, which is very close to the actual situation of financial market in papers. In Figure 4(d), the autocorrelation function of absolute return decays slowly, reflecting the long-memory characteristics of the volatility.

To further verify the statistical characteristics of the power exponent, we repeat the simulation process 1000 times; the mean value of the power exponent is 3.16 and the standard deviation is 0.23. To test the volatility clustering effect of returns, we also use the Breusch-Godfrey test to construct Lagrange multiplier statistics to test for heteroscedasticity. For a given significance level of 0.05, the p-value of the return is 1.665e-15. Thus, we can reject the null hypothesis that the residual variance is constant and deduce that heteroscedasticity does exist which confirms the graphic inference (Figure 4(b)) and is consistent with the empirical results in Section 2.

4.3. Jumps and Structural Changes of the Pricing Process. For the simulation results, we use the LM test to construct the jump statistic based on volatility and use this statistic to test the jump phenomenon of price series.

FIGURE 6: *Simulation results of price series and value series under different thresholds*. (a) The trajectory of price and value with a threshold of 1.5. (b) The trajectory of price and value with a threshold of 2. (c) The trajectory of price and value with a threshold of 2.5. (d) The trajectory of price and value with a threshold of 3.

The LM test is carried out for 1000 groups of data obtained after 1000 repetitions of the simulation process; the average of the 1000 test results is close to the LM test results for the empirical data. Figure 5 is the distribution of jump statistics for a group of simulated data. Comparing the test statistic with the horizontal reference line of the significance level threshold in Figure 5, we can see that a total of 32 jumps have taken place in the 5-minute sampling interval. Compared with the test results of the SSE 50 index in Figure 2, the results simulated by our model are close to the real data.

To simulate the impact of special exogenous information that will lead to the phenomenon of structural changing of the price, we adjust the parameters related to the impact degree of external events. The price changes depend mainly on the value of the assets; that is, the value of the assets as predicted by fundamental investors will lead the price trend. We simulate the structural changes of prices by increasing the threshold and degree of the impact of external events on fundamental investors. We set the threshold θ_2 of the impact of external events that affects the value of assets to be 1.5, 2, 2.5, and 3, which denotes that the probability of occurrence and degree of impact on the expectation of fundamental investors are reduced. The results are shown in Figure 6.

In order to check whether there are structural changes in volatility and compare with the empirical test results, we simulate 5,000 minutes of closing price data with a threshold

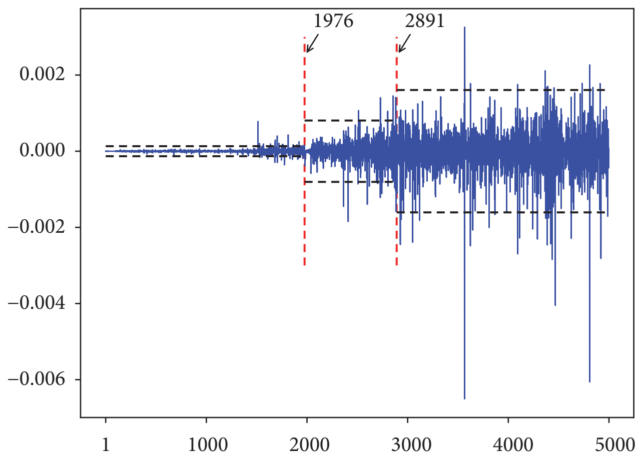

Figure 7: *Structural changes test results of minute returns of simulated data*. The CUSUM test finds 2 structural changes, namely, the 1976 time unit and the 2891 time unit marked by arrows in the figure, which can divide the entire return series into 3 phases. The first phase is from 1 to 1976 time unit, the second phase is from 1977 to 2891 time unit, and the third phase is from 2892 to the end.

of 3 and perform natural logarithmic processing. The return series can be obtained by first order difference. Then, the CUSUM test based on nonparametric estimation is used to detect the structural changes of volatility of in 1-minute return series. The results are shown in Figure 7.

Compared with the test results of empirical data in Figure 3, we find that the price series simulated using our proposed mechanism indeed exhibit the structural change characteristics similar to that of empirical data through the adjustment of parameters. As seen from Figures 6 and 7, by adjusting the threshold value of the model, asset-price process can appear to have obvious jumps and structural changes.

4.4. Summary. In this section, the price dynamics model derived in Section 3 is simulated and analyzed, and the statistical results are compared with the empirical results in Section 2. Referring to the parameter settings in Inoua's paper and the statistical results of empirical data, we adjust and test the parameters, analyze the simulation results, and find that the pricing process has power-law distribution and volatility clustering phenomenon, which is consistent with the simulation results of Inoua's model and similar to the empirical results in Section 2 in terms of price change path, returns distribution, and statistical indicators. Further, LM test is used to test the jump phenomenon of price, and the test results are consistent with the empirical data in Section 2. Finally, by adjusting the parameters, we find that the price dynamics model can also simulate the structural changes existing in the empirical data.

In conclusion, the asset-price process we give displays the typical characteristics of real asset price changes. This provides an important basis for us to understand the inherent laws of asset price changes and improve investment decisions.

5. Conclusion

In this paper, based on the typical characteristics of the power-law distribution, volatility clustering, jumps, and structural changes in China's stock market, we improve and expand the model given by Inoua [11] and propose a price generating mechanism model with exogenous information impact. Our simulation results show that the asset price dynamics model displays the typical characteristics of real asset-price process. It can not only reproduce the power-law tail characteristics and the volatility clustering of returns but also reflect the jumps and structural changes that occur in the actual pricing process.

This result indicates that an asset-price process can follow simple rules, and the reason for its complicated changes may be the impact of external information on the market investors. This helps us to understand the law of price changes of risky assets and make better investment decisions.

Data Availability

The simulation data [CSV format] used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

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References

[1] X. Gabaix, P. Gopikrishnan, V. Plerou, and H. E. Stanley, "A theory of power-law distributions in financial market fluctuations," *Nature*, vol. 423, no. 6937, pp. 267–270, 2003.

[2] X. Gabaix, P. Gopikrishnan, V. Plerou, and E. Stanley, "A unified econophysics explanation for the power-law exponents of stock market activity," *Physica A: Statistical Mechanics and its Applications*, vol. 382, no. 1, pp. 81–88, 2007.

[3] R. C. Merton, "Option pricing when underlying stock returns are discontinuous," *Journal of Financial Economics*, vol. 3, no. 1-2, pp. 125–144, 1976.

[4] B. Eraker, "Do stock prices and volatility jump? Reconciling evidence from spot and option prices," *The Journal of Finance*, vol. 59, no. 3, pp. 1367–1403, 2004.

[5] S. S. Lee, "Jumps and information flow in financial markets," *Review of Financial Studies* , vol. 25, no. 2, pp. 439–479, 2012.

[6] K. Boudt and M. Petitjean, "Intraday liquidity dynamics and news releases around price jumps: Evidence from the DJIA stocks," *Journal of Financial Markets*, vol. 17, no. 1, pp. 121–149, 2014.

[7] B. Eraker, M. Johannes, and N. Polson, "The impact of jumps in volatility and returns," *Journal of Finance*, vol. 58, no. 3, pp. 1269–1300, 2003.

[8] A. E. Biondo, A. Pluchino, and A. Rapisarda, “A multilayer approach for price dynamics in financial markets,” *European Physical Journal Special Topics*, vol. 226, no. 3, pp. 477–488, 2017.

[9] C.-S. J. Chu, M. Stinchcombe, and H. White, “Monitoring structural change,” *Econometrica*, vol. 64, no. 5, pp. 1045–1065, 1996.

[10] T. Mikosch, J. Kreiß, R. A. Davis, and T. G. Andersen, *Handbook of Financial Time Series*, Springer, Heidelberg, Germany, 2009.

[11] S. Inoua, “The Random Walk behind Volatility Clustering,” 2016, https://arxiv.org/papers/1612/1612.09344.

[12] X. Gabaix, P. Gopikrishnan, V. Plerou, and H. E. Stanley, “Institutional investors and stock market volatility,” *The Quarterly Journal of Economics*, vol. 121, no. 2, pp. 461–504, 2006.

[13] S. S. Lee and P. A. Mykland, “Jumps in financial markets: A new nonparametric test and jump dynamics,” *Review of Financial Studies* , vol. 21, no. 6, pp. 2535–2563, 2008.

[14] B. Chen and Y. Hong, “Testing for smooth structural changes in time series models via nonparametric regression,” *Econometrica*, vol. 80, no. 3, pp. 1157–1183, 2012.

[15] D. Kristensen, “Non-parametric detection and estimation of structural change,” *The Econometrics Journal*, vol. 15, no. 3, pp. 420–461, 2012.

[16] K.-L. Xu, “Testing for structural change under non-stationary variances,” *The Econometrics Journal*, vol. 18, no. 2, pp. 274–305, 2015.

[17] E. E. Peters, “Fractal market analysis: applying chaos theory to investment and economics,” *Chaos Theory*, vol. 34, no. 2, pp. 343–345, 1994.

[18] E. Samanidou, E. Zschischang, D. Stauffer, and T. Lux, “Agent-based models of financial markets,” *Reports on Progress in Physics*, vol. 70, no. 3, pp. 409–450, 2007.

[19] N. T. Chan, B. LeBaron, A. W. Lo et al., “Agent-based models of financial markets: a comparison with experimental markets,” *SSRN Electronic Journal*, vol. 70, no. 3, pp. 409–450, 2000.

[20] L. S. Lima and L. L. Miranda, “Price dynamics of the financial markets using the stochastic differential equation for a potential double well,” *Physica A: Statistical Mechanics and its Applications*, vol. 490, pp. 828–833, 2018.

[21] D. Provenzano, “An artificial stock market in a system dynamics approach,” 2013, https://www.researchgate.net/publication/228558031.

[22] S. Inoua, “Speculation and power law,” *SSRN Electronic Journal*, 2017, https://www.researchgate.net/publication/311926269.

[23] H. Kesten, “Random difference equations and renewal theory for products of random matrices,” *Acta Mathematica*, vol. 131, pp. 207–248, 1973.